MATHEMATICAL THINKING

Learning Materials Workshop Blocks and The National Council of Teachers of Mathematics Standards

A Teacher's Guide Pre-K-2

By Mary Gemignani and Alice Leeds

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and

The National Council of Teachers of Mathematics Standards

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Contents

I. Foreward: Geometry and Blocks

Juanita V. Copley, Ph.D.

Geometry and Blocks

Blocks provide many opportunities for young children to experience basic geometric principles. Manipulating three-dimensional objects allows children to discover the attributes of these objects, better understand the two-dimensional shapes that are on the faces of these objects, and learn about symmetry, transformation, and spatial thinking.

Why geometry as part of mathematics?

- Children who have strong spatial sense do better at mathematics (Clements, 1999; Fennema & Sherman, 1977; Guay & McDaniel, 1977; Wheatley, 1990).
- Some students' capabilities with geometric and spatial concepts exceed their numerical skills. If the connection between geometry and numbers can be facilitated, perhaps these children can better learn numbers and other mathematical concepts (Razel & Eylon, 1991, as cited in Clements, 2004).

What does the National Council of Teachers of Mathematics say about geometry and blocks?

The 2000 *Principles and Standards for School Mathematics* proposes that young children (Pre-K–2) spend as much time with geometry concepts as they do with numbers and operations.

The *Standards* list the geometry expectations for Pre-K–2 children as:

- Recognize, name, build, draw, compare, and sort two- and three-dimensional shapes
- Describe attributes and parts of two and three dimensional shapes
- Investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.
- Describe, name, and interpret relative positions in space and apply ideas about relative position.
- Describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance.

- Find and name locations with simple relationships such as near to and in coordinate systems such as maps.
- Recognize and apply slides, flips, and turns.
- Recognize and create shapes that have symmetry.
- Create mental images of geometric shapes using spatial memory and spatial visualization.
- Recognize and represent shapes from different perspectives.
- Relate ideas in geometry to ideas in number and measurement.
- Recognize geometric shapes and structures in the environment and specify their location.

What do international tests say?

The recent TIMSS (Third International Mathematics and Science Study) data indicated that:

- U.S. seventh and eighth graders scored at or slightly below the international average on geometry.
- U.S. fourth and fifth graders scored significantly lower than that the international average on the geometry questions dealing with three-dimensional objects.
- Generally, U.S. students scored at or near bottom in every geometry task.
- Items dealing with three-dimensional objects required that –
    - students predict the result when a three-dimensional five-sided figure was rotated.
    - students name the number of edges or faces on a pictured cube.

Comparisons are the same among preschoolers from a variety of countries. Four-year-olds in America scored 55% compared to 84% for those from China on geometry tasks. (Starkey et al., 1999, as cited in Clements, 2004).

What should young children learn in geometry and spatial thinking?

- Shape concepts begin forming in the preschool years and stabilize as early as age 6 (Gagatsis & Patronis, 1990; Hannibal & Clements, 2000, as cited in Clements, 2004). Children between the ages of 3 and 6 should learn the attributes of both two- and three- dimensional shapes so that they may identify a wide range of examples and non-examples of a wide range of geometric figures.

- Clements (2004) recommends two general areas for geometric thinking.
- Shape and Transformation: 2-dimensional figures, angle, 3-dimensional figures, congruence, symmetry, transformations, composition and decomposition.
- Spatial Thinking: spatial orientation, maps and navigation, and spatial visualization and imagery.

What about the processes involved in block building? How do they relate to mathematical understanding?

NCTM's Principles and Standards for School Mathematics identify five process standards in mathematics for all ages: Problem-Solving, Reasoning and Proof, Communication, Connections, and Representation. (See p. 15/16) The process standards all relate to block building in some way and are identified in the investigations and activities that are included in this guide.

What does research tell us about young children and their interactions with geometric concepts?

- About two-thirds of the interactions Kindergarten children had with their teachers were only to elicit and verify prior knowledge of shapes and matching them. New content or the development of new knowledge about geometric ideas was not encountered in Kindergarten (Thomas, 1982, as cited in Clements, 2004).
- Current practices in primary grades promote little conceptual change. Over time (from first grade to third grade), children were LESS likely to notice attributes (vertices or sides) of shapes given the conventional instruction of geometry in the elementary grades (Lehrer, Jenkins, and Osana, 1998, as cited in Clements, 2004).
- We know that prototypical examples of two-dimensional shapes were easily identified by young children. However, other unusual shapes were not identified and described (Clements, 2004).
- Children use plane figure names for solids. That may indicate a lack of discrimination between two and three dimensions. Learning only plane figures in textbooks during the early primary grades may cause some initial difficulty in learning solids (Carpenter et al., 1976).
- Under the right conditions, children of all ages can apply similarity transfor-

mations to shapes. (Sophian & Cosby, 1998; Confrey, 1992, as cited in Clements, 2004).

- Preference for vertical symmetry develops between 4 and 12 months of age (Bornstein et al., 1981, as cited in Clements, 2004).

Why use Learning Materials Workshop blocks?

- The design of LMW blocks provokes specific questions about geometry and spatial relations, as well as exploring concepts of patterns and measurement, and numbers and operations. Because of the unique shapes and design features of LMW blocks, many problem situations present themselves to students, and more can be created to encourage further investigations. These unique design features include:
  - asymmetrical arches that form a mirror symmetry when paired [Arcobaleno]
  - color placed on the side of the cubes, bobbins, and pyramid shapes that can indicate spatial position
  - pliable tubing that allows for transformation of shapes
  - flexible Mylar that reflects and transforms shapes
  - acrylic domes that magnify forms
  - acrylic prisms that refract light

More research is needed to describe, explain, and develop children's approaches to three-dimensional figures (Clements, 2004).

- Many, if not all, of the geometry standards address expectations that should require three-dimensional blocks. Unfortunately, most examples found in curricula are only given with two-dimensional shapes.
- Students do not perform well with three-dimensional shapes. Most intermediate-grade students have difficulty naming solids (Carpenter, Coburn, Reys, and Wilson, 1976)
- Congruence and transformations are also important ideas for young children but are seldom if ever studied using three-dimensional shapes.
- Symmetry is also important to a young child. This concept is easily seen when children use three-dimensional blocks (see picture of children using blocks).

- Children need to learn to both read and make maps meaningfully. Over reliance on literal pictures and icons may hinder understanding of maps (Downs, Liben, and Daggs, 1988, as cited in Clements, 2004). Perhaps the three-dimensional blocks used as building structures could be an intermediate step before the two-dimensional shape pictures.
- Teachers MUST provide materials and structure the environment appropriately to encourage students to explore both two-dimensional and three-dimensional shapes. They must see many examples of typical three-dimensional shapes (e.g., the rectangular prisms, cubes, and arches in typical block centers) as well as a variety of shapes that are non-examples of the more typical figures.

How should teachers present and develop mathematical investigations with LMW blocks?

There is no one right way to have children work with LMW blocks. Students should have ample opportunity to explore these materials on their own. However, it is important to pose interesting tasks, to begin conversations with children as they work with the materials, and to offer extensions for those children who are particularly involved.

II. INTRODUCTION

Learning Materials Workshop Blocks and the NCTM Standards

Students of all ages enjoy exploring with blocks and inventing novel structures and patterns. Learning Materials Workshop (LMW) Blocks are designed to nourish this aesthetic and creative impulse as an integral part of cognitive growth. The variety of vibrant colors, natural wood grain patterns, geometric shapes, symmetrical and asymmetrical forms, and modular, interchangeable units encourages students to explore the mathematical world and to experience the close relationship between aesthetic and mathematical ideas.

The National Council of Teachers of Mathematics has developed a set of process and content standards that are redefining the nature of mathematics instruction. The Pre-K–2 NCTM Standards are stated briefly below. They all relate to block building in some way and will be identified in the activities.

Content Standards (Content that children should learn)

- Numbers and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

Process Standards (Ways of acquiring and using content knowledge)

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

A detailed explanation of these standards can be found on pp. 14-16 on the NCTM Web site, www.nctm.org. In this guide we look specifically at ways the blocks can be used to support the Pre-K–2 NCTM Standards. The sample investigations are intended to support math programs rather than serve as an all-inclusive curriculum. They can and should be adapted to suit the needs of individual classrooms. This introduction was created to help incorporate the materials into a math program.

Problem Solving

LMW blocks clearly lend themselves to problem-solving activities and to design tasks in particular. Problem-solving situations provide unique opportunities for students to show strengths that pass by unnoticed in traditional academic assignments.

The suggested investigations are open-ended; they can be solved in a number of ways. Encourage students to seek, share, and accept various solutions for each investigation. Conversations will occur quite naturally when students work collaboratively. When using these blocks you might like to:

- Call particular attention to unique, efficient, or sophisticated solutions.
- Ask students to articulate their thinking process by explaining how they solved a particular task and why they used that approach.
- See if students can discover a pattern, a generalization, or a rule based on a discovery in working with the materials. Can they see this generalization applied somewhere else in their environment? For example, if students generalize that a tower needs a sturdy base, have they seen a tower designed with this feature?
- Encourage students to connect their solutions to a structure they have seen in the real world or to other kinds of math they have worked on.
- Help students find ways to extend a challenge further. For example, a multistory building might have the added challenge of needing a watch tower or stilts.
- Encourage students to create their own challenges.

Developmental Levels

The challenges and investigations in this guide are designed for students in Pre-K through second grade. The open-ended characteristics of these blocks allow children to use them on their own level and to be stimulated to stretch their capabilities. Younger children will need to have the challenges read to them. Their documentation will include more drawing. They can also dictate their theories and working procedures to an adult or older child. Older children will add their own written documentation and will work on more complex drawings, charts and diagrams, and extensions.

Time and Space/Classroom Management

The way a teacher introduces materials and schedules their use has considerable impact on student success. To encourage respect for the LMW blocks, make a point of introducing them as a unique set of materials. Find a special place for them in your classroom where they can be easily taken out and put away by the students. Before asking students to focus on a given investigation, provide them with the opportunity to explore the materials. This will promote a broad approach and also help prevent management issues.

The LMW investigations can be used in a variety of formats. One way is to create a center with just one set of materials and one or two investigations. Another way is to engage the entire class, with student pairs working with a particular material and investigation. In the latter case, the teacher will have more time to observe. The teacher will also need to arrange a system for students to rotate from one set of materials to the next.

With any arrangement, it is essential that students have ample time to explore the materials freely as well as to complete their more structured investigations.

Integration

Through integration, students can connect math investigations to other kinds of math, to other subject areas, or to real-world situations.

The investigations themselves provide situations in which students blend such topics as geometry and numeration, measurement concepts and spatial understanding, and probability and estimation.

Language skills blend easily with the investigations. Young children often engage in imaginative play about their structures. Students can be encouraged to discuss their solutions with cooperative partners. They can extend the task to a writing project in which they either explain their work or develop a story based on the solution. Students might write a journal entry about a solution they are proud of or make up a story about their structure and share it with the class.

Art can be incorporated as an important aspect of the process and as a valuable skill for communicating ideas. Teachers can show the value of this skill by providing students with ongoing opportunities to learn and to practice drawing techniques.

Although these investigations have been developed to meet the NCTM

Standards, the LMW blocks can also be used to address national science standards. They are of particular use in science units stressing inquiry and physical science, especially as it relates to the properties of objects and materials.

An investigation can be meshed with a current classroom unit that has real-world implications. When studying space exploration, for example, students can be challenged to design the tallest possible launch pad from the blocks. The Framebuilder investigations would be appropriate when studying homes and houses, while the Colorframes would work well when exploring the attributes of color. The Pyramid could support a unit on ancient civilizations.

Cooperative Learning and Combining LMW Block Sets

LMW blocks are well suited to cooperative learning. The investigations included here work best for one or two students at a time. However, you might choose to combine various LMW sets in addressing a challenge. This is highly recommended, and we have provided an example of how it might be done toward the end of this guide. In combining sets, larger groups of students can work together. Students should be encouraged to observe one another's solutions to help develop their own ideas.

Special Needs/Gifted/ESL Students

Special education students and students who do not have facility in English often have gifts that go undetected by more traditional teaching techniques. LMW blocks give these students an opportunity to display their achievement. These investigations provide teachers and other students with an opportunity to see special needs students in a different light. Additionally, open-ended tasks provide fertile ground in which gifted students can develop sophisticated or unique solutions and extensions. Students who are kinesthetically inclined can find motivation and success with these materials.

Vocabulary

Preview investigations before assigning them to be sure the vocabulary is clear to students. It is important that children understand these concepts in order to do the activities in the book. A list of the math vocabulary words follows:

angle	**dimension**	**pattern**	**square**
arch	**dome**	**pentagon**	**structures**
asymmetrical	**estimate**	**prism**	**symmetrical**
balance	**hexagon**	**pyramid**	**triangle**
circle	**maze**	**rectangle**	**weight**
closed shape	**multi- (as a prefix)**	**solid**	
cube	**notched**	**sort**	
design	**octagon**	**spiral**	

III. NCTM Pre-K–2 Standards

Content Standards (Content that students should learn)

- Numbers and Operations
  - Understand numbers, ways of representing numbers, relationships among numbers, and number systems
  - Understand meanings of operations and how they are related to one another
  - Compute fluently and make reasonable estimates
- Algebra
  - Understand patterns, relations, and functions
  - Represent and analyze mathematical situations and structures using algebraic symbols
  - Use mathematical models to represent and understand quantitative relationships
  - Analyze change in various contexts
- Geometry
  - Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
  - Specify locations and describe spatial relationships using coordinates, geometry, and other representational systems
  - Apply transformations and use symmetry to analyze mathematical situations
  - Use visualization, spatial reasoning, and geometric modeling to solve problems
- Measurement
  - Understand measurable attributes of objects and the units, systems, and processes of measurement
  - Apply appropriate techniques, tools, and formulas to determine measurements

- Data Analysis and Probability
  - Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
  - Select and use appropriate statistical methods to analyze data
  - Develop and evaluate inferences and predictions that are based on data
  - Understand and apply basic concepts of probability

Process Standards (Ways of acquiring and using content knowledge)

- Problem Solving
  - Build mathematical knowledge through problem solving
  - Solve problems that arise in mathematics and in other contexts
  - Apply and adapt a variety of appropriate strategies to solve problems
  - Monitor and reflect on the process of mathematical problem solving
- Reasoning and Proof
  - Recognize reasoning and proof as fundamental aspects of mathematics
  - Make and investigate mathematical conjectures
  - Develop and evaluate mathematical arguments and proofs
  - Select and use various types of reasoning and methods of proofs
- Communication
  - Organize and consolidate mathematical thinking through communication
  - Communicate mathematical thinking coherently and clearly to peers, teachers, and others
  - Analyze and evaluate the mathematical thinking strategies of others
  - Use the language of mathematics to express mathematical ideas precisely
- Connections
  - Recognize and use connections among mathematical ideas
  - Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
  - Recognize and apply mathematics in contexts outside of mathematics

- Representation
  - Create and use representation to organize, record, and communicate mathematical ideas
  - Select, apply, and translate among mathematical representations to solve problems
  - Use representations to model and interpret physical, social, and mathematical phenomena

Standards \ Challenge	ARCOBALENO				COLORFRAMES						PRISMATIC						
	balance	patterns	spirals	design	shapes	estimation	towers	fencing	sorting	design	patterns & squares	shape	symmetry	triangles	stacking	probability	container
1. numbers and operations					✔	✔		✔	✔		✔	✔	✔	✔	✔		
2. algebra	✔	✔	✔	✔		✔			✔	✔	✔		✔	✔			✔
3. geometry	✔	✔	✔	✔	✔	✔	✔	✔		✔	✔	✔	✔	✔	✔		✔
4. measurement						✔	✔	✔							✔		
5. data analysis and probability						✔	✔									✔	
6. problem solving	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
7. reasoning and proof	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
8. communication	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
9. connections	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
10. representation						✔	✔						✔		✔		

Standards / Challenge	CUBES/BOBBINS/ BEAMS			CURVILINEAR			FRAMEBUILDER			PYRAMID			STACKBUILDER		
	bobbin machine	fences	design	machine	chain	design	squares	stacking frames	design	angles & polygons	pyramid	design	color balance	rectangular solid	design
1. numbers and operations		✔					✔							✔	
2. algebra	✔			✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
3. geometry	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	
4. measurement		✔	✔				✔		✔	✔				✔	✔
5. data analysis and probability															
6. problem solving	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
7. reasoning and proof	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
8. communication	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
9. connections	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
10. representation		✔													

V. Challenges and Investigations

BALANCE

Design a structure that looks like it shouldn't balance but it does.

Conversation Starters and Further Explorations:

- Why do you think this works?
- Change it so that it doesn't work.
- Can you turn the pieces over and have it still work?

Extensions and Notes:

- Talk about balance.
  What does it mean for something to balance?
  Help students discover the center of balance.
- These materials can balance in some unusual ways.
  Make sure that your students have explored this thoroughly.
  Encourage students to build vertically and to incorporate all of the pieces.
- Encourage students to use visual measurement.

NCTM Standards Addressed:

- Geometry
- Problem Solving
- Reasoning and Proof
- Communication
- Connections

PATTERNS

Make as many different patterns as you can.

Explore both symmetrical and asymmetrical patterns.

Conversation Starters and Further Explorations:

- Create a "cave" where the purple piece is hidden.
- What natural objects do your patterns remind you of?
- Can you make both symmetrical and asymmetrical patterns?
- If you have a symmetrical structure, can you break it in half and have it still stand up?

Extensions and Notes:

- How are arches and circles used in architecture?
- Encourage students to build symmetrical designs both horizontally and vertically.
- Use the Mylar. See how the structures can be changed.
- Try to make the same design/structure both standing up and lying down.
- Challenge the student to make other changes in the orientation of pieces.

NCTM Standards Addressed:

- Numbers and Operations
- Algebra
- Geometry
- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations

SPIRALS

Make a spiral design.

Conversation Starters and Further Explorations:

- Make a curved line design.
- Make another kind of spiral using all the blocks.
- Make two spirals that are connected.
- Where can you find spirals in the world around you?

Extensions and Notes:

- Read about spirals with your students.
- Help your students discover spirals in nature (shells, fiddlehead ferns). Help them find spirals in architecture and other human-made objects (screws, staircases).
- Bring in pictures of spirals.

NCTM Standards Addressed:

- Algebra
- Geometry
- Problem Solving
- Reasoning and Proof
- Communication
- Connections

DESIGN

Try one of these tasks:

- Use the tray in as many structures as you can.
- Design an archway.
- Design a cave.
- Make a maze.
- Design a playground for a mouse.

Conversation Starters and Further Explorations:

- How does the tray affect what you can do with the materials?
- Invent something and tell how it is used and how it helps people.
- What else can you design with these materials?
- Choose your favorite structure. Draw a picture of it. Label your drawing.

Extensions and Notes:

- Build the same structures with other materials.
  Compare the properties of the different building materials.
- Ask your students to make a design task for other students.
- For some children, putting the set back into the tray is a challenging puzzle.

NCTM Standards Addressed:

- Algebra
- Geometry
- Problem Solving
- Reasoning and Proof
- Communication
- Connections

SHAPES

Make a three-sided shape where all the sides are the same size.

Make a three-sided shape where two sides are the same and one side is longer.

Make a rectangular shape.

Make a square.

See what other shapes you can make. How many sides do they have?

Can you make a perfect circle?

Conversation Starters and Further Explorations:

- Which shapes are the most sturdy?
- Make a square shape with a triangular shape on top.
- What objects in the real world do your shapes remind you of?

Extensions and Notes:

- Encourage the students to make the shapes in a variety of sizes.
- Review the names of the geometric shapes.

NCTM Standards Addressed:

- Number and Operations
- Geometry
- Problem Solving
- Reasoning and Proof
- Communication
- Connections

COLORFRAMES

ESTIMATION: AREA AND PERIMETER

Estimate how many frames you would need to cover this page.

Check to see how close you were.

Conversation Starters and Further Explorations:

- How could you find the area without covering the entire page with Colorframes?
- What if the frames were lying on their sides? Standing up?
- Find the perimeter and area of other things using the Colorframes.
- Use the frames to find the length of a table. How about your classroom? Your height?
- If you lined up all the frames, how far across the room do you think they would go? Try it out and see! Measure the length.
- Find a variety of different objects in the room and graph them by size.
- Use other LMW materials to find area and perimeter.

Extensions and Notes:

- One technique for accomplishing this task is to cover a fractional portion of the paper and multiply accordingly. For example, cover half and multiply by two.
- Another technique is to determine how many rows and columns there are, and then multiply.
- Introduce the concepts of area, perimeter, multiplication.
- For younger children, counting skills can be reinforced.

NCTM Standards Addressed:

- Number and Operations
- Geometry
- Measurement
- Data Analysis and Probability
- Problem Solving
- Communication
- Connections
- Representation

TOWERS

Build several towers.

Experiment with several designs and different kinds of bases.

Which design is the tallest? Which tower is the most sturdy?

Which design could hold the most weight?

Conversation Starters and Further Explorations:

- How could you prove which one was the strongest? Test your theory.
- Guess how tall it is. Measure your tallest tower.
- Does the tallest tower always have the most blocks?

Extensions and Notes:

- Explain to students that sometimes buildings need to withstand high winds or earthquakes. Have students test the strength of their tower by building it on the base plate and gently tilting or shaking the plate. You might want to bring in a large electric fan to see how much wind their structure can withstand.
- Use different materials to build the same tower.
- Have all of the children in your class make a tower, measure them, and make a graph comparing the different heights.
- Talk about the foundation of a building and why it is important.
- This is a game for two children. Each child uses the blocks of his/her choice and creates two towers. On each turn s/he rolls a die, counts out that number of blocks and begins making one stacking tower. On each turn, the child may build on either tower as long as s/he uses the exact number of blocks rolled. After five rolls each, each player counts the number of blocks in his/her two towers and then compares the height. Each child then describes his/her "winning" tower to the other player using number words and measurement comparison terms (taller, fatter, wider, etc.).

NCTM Standards Addressed:

- Number and Operations
- Geometry
- Measurement
- Data Analysis and Probability
- Problem Solving
- Communication
- Connections
- Representation

FENCING

Use 12 frames to build a fence.

How many different ways can you do this?

Make a drawing of each.

Conversation Starters and Further Explorations:

- Try this task with 16 or 25 frames. What shape has the largest area?
- Measure the sides of each fence. Add these dimensions to your drawings.
- Use Prismatic or inch cubes to measure the area inside the fence.

Extensions and Notes:

- Help the students discover that squares give the most area, given a constant perimeter.
- This task could also be done with different materials such as tiles, tubes, or straws.

NCTM Standards Addressed:

- Number and Operations
- Problem Solving
- Reasoning and Proof
- Communication
- Connections

SORTING

Show several ways that you can sort the frames into groups by color.

Conversation Starters and Further Explorations:

- How many sets do you have? How many frames do you have in each set?
- How is each group different from the rest?
- What familiar colors do you see?
- Which frames are darker? lighter? brighter? duller?

Extensions and Notes:

- Colors have three attributes: hue (color name), value (how dark or light they are), and saturation (intensity: how bright or dull they are).
- See *The Learning Materials Workshop Blocks* by George Forman and Karen Hewitt, pages 66-67.
- Invite your art teacher or an artist from the community to present a lesson on color theory.
- Try mixing paints to create some of the colors on the tiles. Measure the amount of paint needed to make the new color.

NCTM Standards Addressed:

- Number and Operations
- Problem Solving
- Reasoning and Proof
- Communication
- Connections

TRACING SHAPES

Make a structure. Trace the bottom of the structure and look at the shape that is outlined on the paper. Can you make...

- A triangle where all the sides are the same size?
- A triangle where two sides are the same size?
- A triangle where no two sides are the same size?
- A rectangle where only two sides are the same size?
- A square using at least eight blocks? Fourteen blocks?
- An octagon?
- A circle?

Conversation Starters and Further Explorations:

- What traced outlines were easiest to make? Why do you think so?
- What traced outlines are impossible to make? How do you know?
- Is it possible to make a rectangle shape using only four blocks?
- Can you make a cube with the Colorframes? What about a triangular or rectangular prism?

Extensions and Notes:

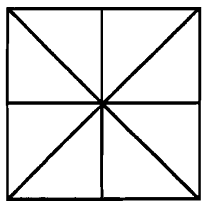
- Structures made can be turned and other faces traced. Compare those faces with the ones made previously.
- Look at the diagrams on page 84 for the names of the Polyhedra – three-dimensional solid shapes with flat faces.

NCTM Standards Addressed:

- Geometry
- Measurement
- Problem Solving
- Communication
- Representation

PATTERNS AND SQUARES

Make a square out of triangular faces using eight prisms.



Choose two colors and make a square pattern. Using the same prisms, record how many ways you can vary the pattern.

Conversation Starters and Further Explorations:

- How do you know you have made a pattern? Could you read your pattern to me?
- What if some of the Prismatic blocks were covered up? Could you tell what blocks are underneath?
- What is the largest square you can make?
- What if you used nine prisms... 10 prisms... 12 prisms... 16 prisms...? Could you still make a square on the faces? Why do you think so? Try it!
- What if you only had two prisms... four prisms... six prisms...? Could you still make a square on the faces? Why do you think so? Try it!
- If you only had two prisms and you put them together, could you make any other shapes?
- What if you had eight prisms? Could you make a cube?
- How many prisms would you need to make a cube? Show what you have discovered.
- Make a patchwork combining different square patterns into a larger square. Draw it on graph paper. Where have you seen these kinds of patterns?
- How many smaller squares does it include? Now try a three-by-three square. How many smaller squares does it contain? Continue building larger squares. How many triangles are in the square?
- How many of each color are in the square?
- Make a pattern combining smaller squares and large squares.

- Make a pattern in the box when you put the prisms away.

Extensions and Notes:

- Students can copy patterns, continue patterns, or create new patterns. Working on patterns involves many mathematical and artistic processes including spatial relationships, symmetry, order, congruence, tessellations, fractions, ratios, transformational geometry, organizing, and describing ideas.
- Have students take 12 prisms using at least six different colors. How many different patterns can they make? Record them on graph paper.
- Make a scale drawing of a pattern you make with all 72 prisms. Put the drawings in the box to inspire other students to try.
- Bring in a book with various patchwork quilt designs to share with your students. Have them try to replicate some of the designs using the prisms.

NCTM Standards Addressed:

- Algebra
- Geometry
- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

SHAPE

See how many shapes you can make.

Can you make a hexagon, a pentagon, a parallelogram?

Conversation Starters and Further Explorations:

- How many sides does the prism have?
- Take a prism and draw around each side.
  Does each side have the same shape?
  What shapes did you make?
- Take two prisms from the box.
  How many different shapes can you make with only two prisms?
  Draw around each different shape.
  What are the names of the shapes that you made on your paper?
- Use as many prisms as you need.
  What shapes can you make?
- Make drawings of constructions from different perspectives or angles.

Extensions and Notes:

- Help the children discover that the square base of the prism is congruent with a square made of the triangular sides of four prisms.
- Where in nature and the human-made world are prisms used?
- Discuss the geometric shapes of various crystals. Show pictures of them.
- Students might like to try to create these crystal shapes with Prismatic or other materials (paper, toothpicks and marshmallows, straws and pipe cleaners).
- Discuss the concept of negative space.
  What shapes are made with the negative space?

NCTM Standards Addressed:

- Geometry
- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

PRISMATIC

SYMMETRY

Make symmetrical patterns using the triangular faces of the prisms.

Try adding as many prisms as you can to your pattern.

Create half of a pattern. Invite a partner to complete the symmetrical design. Try to complete the design in various positions (mirrors or Mylar could be used to help).

Conversation Starters and Further Explorations:

- Can you divide your pattern in half? in fourths?
  How many prisms are in the whole pattern? in half? in one-fourth?
  Put it back together and use the Mylar to double your pattern.
  Now divide it into fractional portions and see what happens.
- Give each triangle a value (2¢). How much is your pattern worth? Try giving various pieces different values (clear=5¢, natural=3¢, etc.)
- Build a design with a given value or area. Can you make it symmetrical? (Encourage students to discover that they need an even number of pieces in order to do this.)
- What natural and human-made objects are symmetrical?
- Gather photographs of symmetry from magazines. Create a collage from them.

Extensions and Notes:

- Do a lesson on symmetry.
- Collect examples of geometric design motifs on pottery, fabrics, tiles, etc.
- Take one of the designs and create it using the prisms. See how you might vary it.
- Use the triangle graph paper to copy students' designs.

NCTM Standards Addressed:

- Number and Operations
- Algebra
- Geometry
- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

TRIANGLES

Make triangles of different sizes using the triangular faces of the prisms.

Make a variety of patterns within your triangles.

Conversation Starters and Further Explorations:

- How many small triangles are in each of your larger triangles?
- Place the triangles in order from smallest to largest.
- Do you see a pattern?

Extensions and Notes:

- Younger children may not see the possibilities of creating increasingly large triangles by arranging small triangles in a progressive pattern. You may want to show them this pattern in the form of a short presentation and see if they can replicate it or take it to the next larger triangle.
- It is possible to make triangles representing square number patterns using one, four, nine, sixteen, twenty-five prisms.
  Draw pictures of the triangle patterns.
  Label each drawing with its number equivalent.
  Using square tiles or graph paper, show that these numbers also form squares; in fact, they are called square numbers.
- Explore the shapes you can make on graph paper with other kinds of numbers. All even numbers can make rectangles, for example.

NCTM Standards Addressed:

- Number and Operations
- Algebra
- Geometry
- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

PRISMATIC

PROBABILITY

Look carefully at all of the different kinds of prisms.
Remove the clear prisms and put the rest of the prisms in a container.
Without looking, choose one of the prisms.
Before looking, what color do you think you have chosen?
Return the prisms to the container and shake them all up.
Choose and return prisms 50 times. Which one did you choose the most? Why?

Conversation Starters and Further Explorations:

- Record your results and make a graph.
- Remove all of the red prisms. How does that change your results?
- Now remove all of the natural wood prisms. How does that change your results?
- Put a certain number of prisms in a bag; e.g., four red, eight yellow. Draw a prism out, record its color, and put it back in the bag. After 12 draws, have students guess the number of each color in the bag. Do the entire experiment a second time, allowing students to change their guesses based on second experiment. Discuss theoretical probability of drawing each color (red=4/12, yellow=8/12) and how close the experiments came to that. Activity can be increased to three or more colors. Discuss the need for many trials.

Extensions and Notes:

- Have the students count and record the number of prisms of each type and correlate those with the results.
- Show your students how to tally, chart, and graph their results.
- Emphasize the need for many trials when gathering data. Point out that students who did many trials had data more consistent with each other.

NCTM Standards Addressed:

- Numbers and Operations
- Data Analysis and Probability
- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

BOBBIN MACHINE

Using the rubber bands, invent a machine that makes one bobbin turn another bobbin.

Conversation Starters and Further Explorations:

- Can you make the second bobbin turn in the opposite direction? Can you make a third bobbin turn? What does this remind you of?
- Can you make the machine move a dowel?
- Can you make a machine that makes a sound?

Extensions and Notes:

- Encourage students to see what happens when the rubber bands are twisted.
- Read selections from *The Way Things Work* by David Macaulay and *Wheels That Work* by Bernie Zubrowski to students.

NCTM Standards Addressed:

- Geometry
- Problem Solving
- Communication
- Connections

CUBES, BOBBINS, BEAMS

FENCES

Estimate how many beams are needed to build a fence around the box.

What is the smallest number of dowels you would need to build a fence around the box? What is the largest number?

Build a fence using all the materials.

Conversation Starters and Further Explorations:

- Why might you want to build your fence with larger dowels?
  Why might you want to use smaller dowels?
- Without counting, estimate if there are more dowels or more beams.
- Are there more cubes or more bobbins? Estimate.
- Are there more beams or more cubes? Estimate.
- What do you estimate is the number of holes in all the cubes? Bobbins? Beams? How many holes are there altogether?

Extensions and Notes:

- Explore with your students what a hole is. There is some ambiguity here.
  Are there six holes in the cube or three holes?
- Make a symmetrical fence.
- Students who think about cutting dowels to minimize the number of dowels needed to surround the box could be encouraged to think about fractions and division.

NCTM Standards Addressed:

- Number and Operations
- Geometry
- Measurement
- Problem Solving
- Reasoning and Proof
- Communication
- Connections

SPINNING TOWER AND VEHICLE

Spinning tower: How did you get it to spin? Why might you want it to spin?

Make a vehicle that can carry something. How much weight can it carry?

Conversation Starters and Further Explorations:

- What else can you build from these materials?
- Choose your favorite structure and draw a picture of it. Label your drawing.

Extensions and Notes:

- Build the same structures with other materials.
  Compare the properties of the different building materials.
- Ask your students to design a task for other students. Create a challenge card.

NCTM Standards Addressed:

- Geometry
- Problem Solving
- Communication
- Connections

SQUARES

Create squares inside squares.

Find a way to make them stand up.

Order the squares from smallest to largest. Count them.

Conversation Starters and Further Explorations:

- Find a way to stack squares on top of squares.
- Measure how high you can go.
- Are there other ways you can build this structure?
- Think of a real-world situation where you would need to solve the problem of stacking squares.

Extensions and Notes:

- Discuss balance and the properties of squares, rectangles, and right angles.
- Relate this to the frame of a house. What happens to the stability of the structure when the right angles are lopsided?
- Bring in a carpenter's square and level. Talk about the importance of keeping surfaces level and angles square when building.

NCTM Standards Addressed:

- Geometry
- Measurement
- Problem Solving
- Communication
- Connections

FRAMEBUILDER

STACKING FRAMES

Using the dowels and the frames, find a way to build a pyramid by stacking square frames on top of each other. What does this remind you of?

Conversation Starters and Further Explorations:

- What happens if the dowels are removed?
- How many dowels will you need to remove before the structure falls down?

Extensions and Notes:

- Discuss the concept of balance.
- Provide students with pictures of pyramids.

NCTM Standards Addressed:

- Algebra
- Geometry
- Problem Solving
- Communication
- Connections

DOWEL STRUCTURE

Build a structure with dowels. See if you can remove the dowels. What problems do you run into?

Conversation Starters and Further Explorations:

- Measure your structures.
- What else can you design with these materials?
- Choose your favorite design and draw a picture of it. Label your drawing.

Extensions and Notes:

- Build the same structures with other materials.
- Ask your students to design a task for other students.

NCTM Standards Addressed:

- Geometry
- Measurement
- Problem Solving
- Communication
- Connections

PYRAMID

ANGLES AND POLYGONS

Make a shape with all the angles fitting together.

Try to make a closed shape.

See if you can build an octagon, a hexagon, or a pentagon.

Is this possible? Why or why not?

Conversation Starters and Further Explorations:

- Draw the shapes you made.
- How might you change the angles or lengths to make them work better?

Extensions and Notes:

- Learn the names of different polygons. Draw and label them.
- Talk about the meaning of an angle.
- Some students may recognize that the size of the angle and the length of the side are related, making this an impossible task. Other students will correct for the angle as needed to form a polygon. Both solutions are, of course, acceptable.

NCTM Standards Addressed:

- Geometry
- Problem Solving
- Communication
- Connections

PYRAMID

Make a pyramid.

Now build it upside down.

See if you can make two pyramids from the materials.

Conversation Starters and Further Explorations:

- What is special about these building materials?
- How does the shape of the material help you build these pyramids?
- What other kinds of structures can you build with these materials?
- What kinds of challenges might there have been in building the ancient pyramids?
- How were the ancient pyramids the same as the one you just built? How were they different?

Extensions and Notes:

- Use the book *Pyramids* by David Macaulay to look at different types of pyramids.
- Use sugar cubes, toothpicks and marshmallows, straws, and paper to make pyramids.

NCTM Standards Addressed:

- Geometry
- Problem Solving
- Communication
- Connections

COLOR BALANCE

Using all of the colored pieces, design a symmetrical pattern. With the help of a dowel, stand it upright. What happens when the pieces are moved around the dowel?

Conversation Starters and Further Explorations:

- Develop a variety of patterns. Use graph paper to record them.
- Explore ways symmetrical patterns and space can be used. What do these designs remind you of?

Extensions and Notes:

- Many cultures decorate their clothing and pottery with designs that balance color and geometric patterns. You could bring in examples of Native American and African patterns that show the use of this type of symmetry.
- Visit a museum to look for symmetry and color balance used by peoples around the world.

NCTM Standards Addressed:

- Algebra
- Geometry
- Problem Solving
- Communication
- Connections

RECTANGULAR SOLID

Standing all of the notched pieces on edge, put them together to form a rectangular solid.

Conversation Starters and Further Explorations:

- What pattern did you discover? Can you build it another way? Draw the solid.
- What do you notice about the way the pieces fit together?
- Measure the dimensions of the solid.

Extensions and Notes:

- Draw the rectangular solid on graph paper.
- Make sure that students are aware that each pair equals a common length (e.g., 12 inches) just like math fact families (5+4=9, 4+5=9, 9–4=5, 9–5=4).

NCTM Standards Addressed:

- Number and Operations
- Algebra
- Geometry
- Measurement
- Problem Solving
- Communication
- Connections
- Representation

STACK CONSTRUCTION

Try to build one of the following structures:

- A fence using all the notched pieces. Is it possible to close this fence?
- A double set of stairs
- An interesting pattern

Conversation Starters and Further Explorations:

- What else can you design from these materials?
- Choose your favorite structure and draw a picture of it. Label your drawing.

Extensions and Notes:

- Build the same structures with other materials.
- Ask your students to design a task for other students.

NCTM Standards Addressed:

- Algebra
- Geometry
- Measurement
- Problem Solving
- Communication
- Connections
- Representation

BUILDING PLANS

Build a construction. Make "building plans" of the construction on a large sheet of paper. Remember to write down (or describe) the specific measurements and a materials list. Roll the plan up and store it so that other children can reconstruct the building from the plan.

Conversation Starters and Further Explorations:

- How tall is the building? How will you record that on your plan?
- How wide (fat) is the building? How will you record that on your plan?
- How many blocks did you use? What if you had used three more? What would happen?
- How would someone else know that you used that block?
- How could you make a picture that would show it? How would you show the different shaped blocks on your plan?
- What if you looked at your building from the top? How would your plans look different?

Extensions and Notes:

- Encourage children to draw plans for their buildings from a different perspective. Add these new pictures to the plan.
- Have children talk with peers and get opinions about how their plans could be made better.

NCTM Standards Addressed:

- Algebra
- Geometry
- Measurement
- Problem Solving
- Communication
- Connections
- Representation

COMBINED SETS

FIND MY BUILDING

Look at the photos of children's previous building constructions. Read (or listen to) the descriptions the children made of their block constructions. The descriptions include position words, specific measurements, and geometric vocabulary. See if you can identify the matching photos.

Conversation Starters and Further Explorations:

- How many blocks are in this building? Are you sure that is all? How do you know if any blocks are hiding?
- You used "(a particular descriptor)" when you told me about your building. How would you use that same word to describe something else in this room?
- Are you sure you have identified the correct block from the pictures and words? What if you only had the photo? What if you only had the geometric words? What if you only had the specific measurements? What if you only had the position words?
- Could any of your descriptors be used to identify all of the constructions?… two of the constructions?… three of the constructions?… most of the constructions?

Extensions and Notes:

Begin with the photo and word descriptions and ask children to re-create the structure using those clues. Use only part of the descriptors and ask children to re-create the structure.

NCTM Standards Addressed:

- Geometry
- Measurement
- Communication
- Connections
- Representation

CHALLENGING STRUCTURES

Use the Pyramid blocks to make an upside down pyramid

Use the Arcobaleno blocks to make an upside down volcano.

Build on only one of the Arcobaleno blocks and make a structure that contains at least five arches.

Use both the Pyramid and the Arcobaleno blocks to build on top of each other.

Conversation Starters and Further Explorations:

- Do you think these challenges are possible? Why or why not?
- Describe how you accomplished this challenge.
- What didn't work? How did you fix it?
- Draw the structure that answered the challenge.

Extensions:

Challenge children to come up with challenges of their own for the teacher or for their friend. Ask them to describe why they are challenges.

NCTM Standards Addressed:

- Geometry
- Problem Solving
- Communication

COMBINED SETS

MAP IT

Try some of the tasks below. Make your structure on top of the graph paper. [Use graph paper 10 inches x 10 inches OR 20 cm by 20 cm]. Place your buildings directly on the paper. Remove the blocks and write symbols directly on the graph paper to show where the blocks were placed. The tasks are:

- Draw a river from one corner to the other corner of the map. Build a bridge from a tall house on one side of the bridge to a short house on the other side of the bridge.
- Draw two streets on the graph paper going from north to south and draw two more streets going from east to west (like a checkerboard). Place tall buildings on one street and short buildings on another street. Place medium height buildings on one street and no buildings on another street.
- Separate the graph paper into four sections or "rooms." Put furniture in each room and decide what each room will be. Make a map showing where the furniture should be placed.
- Pretend that the four-by-four square in the center of the grid paper is a lake. Make houses surrounding the lake. Make your "lake resort" so that people can go from one house to the other without going through the lake.

Conversation Starters and Further Explorations:

- How can you make your map understandable so that anyone can understand how to place the blocks?
- How would another builder know what block to place on your map?
- Have you solved the entire problem? What about...?
- Could you build it...draw it another way?
- Describe where the ______________ is in relation to the______________.
- What if I want to go from the small house to the tall house? What would you tell me to do?
- How far is it from ______________ to ______________? How do you know?

NCTM Standards Addressed:

- Geometry
- Communication
- Representation

VI. ASSESSMENT

Overview

The following pages contain sample assessments of open-ended tasks that make use of LMW blocks. The assessment approaches used here can easily be adapted for use with a variety of similar open-ended tasks. We focus on anecdotal and portfolio assessment as we find these the most helpful means of assessing open-ended tasks. In addition, we have shown how students' drawings and photographs of student work can be used as assessment tools.

Why Assess?

The purpose of assessment is to improve learning. It is easy to lose sight of this. Traditionally, a student's evaluation has been viewed as the final judgment of how much a student has learned or accomplished. The assessment models in this book are concerned with ongoing appraisals of what a student knows or can do, with the goal of adapting learning opportunities to promote student growth. Each activity provides numerous opportunities for assessing students. The ideal assessment tool points the way to the next step in teaching and learning. Our aim in assessing a child's work is to answer the question, what next?

One of the best ways to ensure the right connection between instruction and assessment is to embed assessment into activities. When students are involved in this type of assessment, it is a natural part of their learning experience. The LMW blocks yield a multifaceted view of students' understandings and skills. They allow children to demonstrate their achievement while accommodating learning styles that are often overlooked in more traditional assessments. While we certainly do not suggest that you try to assess all the solutions students come up with, the LMW activities provide a fine opportunity to gather assessment data about both academic and social skills. While providing useful information about all students, these data are particularly valuable in the case of students who cannot be assessed in more traditional ways, i.e., special needs and ESL students.

Types of Student Assessment

There are many ways to assess what children are learning. The LMW activities provide a wonderful opportunity for teachers to observe and interact with students to assess their thinking skills and levels of understanding. Teachers can use guided observations, interviews, photos, and children's drawings as a means of assessment. All of these methods can be part of a child's learning portfolio.

Observation

A powerful approach to assessing hands-on activities is to observe students as they carry out their work. It is important for teachers to find ways to make this possible. Observation works best when the teacher can focus on one or two students at a time and make notes on their work and interactions. In some instances, the teacher will gather important information by observing from the sidelines. In other cases, the teacher can gain insight, or promote understanding, by questioning children about their work or by making observations aloud.

It is important when observing not to watch too much. The teacher should determine ahead of time which skills or processes will be assessed. This clarifies the purpose and makes it easier to observe more children in one session. Often a child's approach to a problem will give insight into how the child thinks. Is the child persistent? Does she talk to herself while working? Does she draw on skills she has already mastered?

When observing students, teachers should

- Notice how students interact with both the materials and each other
- Question students about their solution and their thinking process
- Encourage students to push themselves to the next level with questions
- Keep student record sheets in order to document progress

Student Interviews

When children are told ahead of time that the teacher will be talking with them about their work, they become conscious of their own learning and begin to self-assess. When students learn to assess their own work, they become full partners in their own learning process. Children might simply be asked if they are pleased with their solution, how they managed to accomplish the task, what was easy and what was more difficult, or what aspect they would attempt to improve next time. Student interviews can give a picture of a child's thinking and depth of understanding. Ask students to assess their own work. Talking about their work encourages mathematically rich interactions and leads to the fine-tuning of solutions.

Photographs and Drawings

Observation and student interviews can be labor-intensive. There are times when teachers need a more efficient means of documenting a student's work. If a child reaches a solution that she is proud of or that the teacher wishes to reflect upon, a photograph can be taken. This can be pulled out and used again for comparison purposes or as a jumping-off point for a discussion with the student.

Children can also be asked to copy a solution in a drawing. Representational drawing is an important form of communication for young mathematicians. As students develop skill and insight, their drawings reach higher levels of sophistication. Drawing skills need to be presented and practiced as part of the curriculum; the visual representation of ideas is an important form of communication. As children learn to label their drawings with words, numbers, and other mathematical symbols, they are developing skills for the more sophisticated project of doing a complete portfolio write-up. In writing about problems they have solved, students can gain deeper understanding of mathematical processes and concepts. And while articulating mathematical ideas, they are increasing their writing skills. In this process, it is important to emphasize that concise write-ups incorporating strong visual and symbolic information portray mathematical understanding most clearly.

Both drawings and photographs can be assessed and kept in a student's portfolio.

Portfolios

Portfolio assessment is defined as a collection of student work used to determine individual growth over time. A portfolio piece shows how a problem was solved through mathematical language, symbols, and representations. An individual portfolio piece is a figurative snapshot showing where a student is, in relation to the topics and concepts addressed, at a given time. Portfolios are valuable to both students and teachers when together they can view a collection of assessed work and arrive at an understanding of strengths and areas of challenge. Portfolio pieces are also a nice addition when sharing student progress with parents.

Portfolio work can be assessed to determine individual growth in order to plan appropriate learning experiences. When choosing an activity to include in a portfolio, ask the child to write or dictate a description of the solution, accompanied by a visual representation—a labeled drawing, photo, video, or recording.

EXAMPLES OF STUDENT ASSESSMENT: Observations and Interviews

The following pages contain assessment examples of open-ended tasks using LMW blocks. These are merely examples of what might happen when a child attempts to solve a given activity. You may note that we have reworded or revised some of our original tasks to suit a particular form of assessment. We encourage you to do the same when adapting these tasks to meet your students' needs.

SYMMETRY

Make as many different symmetrical patterns as you can.

Context and Connections

Students have been studying patterning and can create a large number of patterns on their own. They have just been introduced to the term and concept of symmetry and have had experiences working with a variety of manipulatives including Unifix cubes, tiles, pattern blocks, and LMW's Prismatic. Students have already used the Arcobaleno in an exploratory way and as a puzzle when putting it back into its container.

What This Task Accomplishes

Students use their reasoning skills to create as many symmetrical designs as they can. Most students will discover that a symmetrical design can be reversed or inverted to create another symmetrical design. This task develops students' skills in seeing the relationships between patterns and helps them make mathematical connections. Because there are two of each shape, the task develops a rudimentary concept of fraction and part-to-whole. When students build both horizontally and vertically they use their spatial sense.

Tips

Make certain that students understand the concept of symmetry. A lesson on symmetry could be an extension of a lesson in patterning. When introducing the concept of symmetry, the teacher may want to use Mylar or a mirror.

Many children solve the problem by building only on a horizontal plane rather than both horizontally and vertically. Students should be encouraged to solve the problem using both planes. If students have difficulty starting, ask them to make a symmetrical building. Then ask them to divide the building in half.

Materials

LMW Arcobaleno block set

Colored pencils, crayons, or markers, and paper for illustration

Time Required for Task

One 45-minute period

Level

Pre-K–2

What to Look For

There are many possible solutions to this task. Students who solve the problem using both a horizontal and a vertical plane are mathematically more mature than those who use only one plane. Students who also solve the problem by creating a symmetrical design basing the arches vertically on a horizontal plane are demonstrating an advanced understanding of symmetry. Rotational symmetry might also be addressed, depending upon the sophistication of the students' understanding.

Anecdotal Notes

Age: 7 years, 2 months

Grade: 1

This child is a special-needs student who has receptive and expressive *language difficulties. She has already done symmetry tasks with tiles and pattern blocks. The child spent the first 15-20 minutes exploring the materials.*

Teacher: Do you remember when we made symmetrical patterns last week?

Student: Yeah…I remember.

T: Do you think you can make symmetrical patterns with these new materials?

S: Uh, huh…I can do a magnet. (Student confuses the word *symmetry* with *magnet.*) I can do that. (She continues to work with the Arcobaleno for several minutes but does not produce a symmetrical pattern.) There.

Teacher realizes that the student does not remember the concept of symmetry and decides to use the time to re-teach.

T: Oh…hmmm…Put your hands like this. (Teacher models with her own hands by placing them on the floor next to each other so that they appear symmetrical.) These are symmetrical. See…This thumb matches with this thumb and this finger with that one, and this one with that one. They are symmetrical.

S: Mmmm.

T: Look at what I have. (Teacher takes out a strip of Mylar.) It looks like a mirror, doesn't it? Look at my hand when I put it on top. I can make it symmetrical. (Teacher places her hand on top of the Mylar.) I can use this to make a symmetrical design with blocks too. Watch. (Teacher demonstrates by making a four-block symmetrical design.)

S: Oh…

T: Do you want to make one? Do you want to make a symmetrical design?

S: Sure. (Student makes a design that is reflected in the Mylar.)

T: (Teacher continues to emphasize the words *symmetry* and *symmetrical.*) What a nice symmetrical design you have made. Do you think you can make a symmetrical design without using the mirror?

S: Yup.

T: Why don't you use some of these blocks. (The teacher hands the student three pairs of matching blocks, thus limiting the possibility of failure.)

S: There!

T: You did it!

S: We did that with the tiles, 'member?

T: Ahh! Yes! You have a good memory...If I make a little design do you think you can make it symmetrical?

S: Yup.

T: OK. Try this one. (Teacher makes a simple horizontal plane design.)

S: (Student copies the design symmetrically.) There!

T: Wow!! Can you make one all by yourself?

S: Sure. (Student makes several more symmetrical patterns.) I can do this.

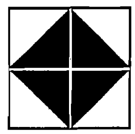
Some Final Notes

This is an example of a situation in which the child does not have the basic concept needed to complete the task. Although the teacher at first felt that the child could do the task, she soon saw that she needed to spend time re-teaching. Note that rather than observing the child, the teacher immediately switched her approach to make use of the teaching moment.

PRISMATIC

PATTERNS AND SQUARES

Choose eight prisms: four of one color and four of another. Make a square color pattern from the triangular faces. An example is shown. How many ways can you vary the square?



Context and Connections

Throughout the year students have been using the LMW blocks for free play and to do mathematical tasks. They have experience with pattern and symmetry. In social studies they studied pioneers and have learned about the history of quilt making and its importance in pioneer life. As a follow-up to this task they will be creating their own quilt squares, first on graph paper, then on paper, and finally with fabric.

What This Task Accomplishes

This task draws heavily on pattern and relationships, design, and spatial sense. It connects nicely as an introductory activity to a social studies unit on pioneers. Reasoning and problem solving are important in discovering the many different ways two colors can be combined.

Tips

For students who are having difficulty, use inch-blocks of two colors. For more advanced students, this task can be expanded by increasing the number of prisms or the number of colors used.

Be aware of how students are defining the word "pattern." Some may define it as a repeating or symmetrical pattern within the eight prisms; others may define it as any configuration of the prisms that would later be repeated and connected to another set of eight prisms. Different definitions are fine as long as the student can clearly articulate or show his/her concept of pattern.

Materials

LMW Prismatic block set

Colored pencils, crayons, or markers and graph paper

Time Required for Task

One 60-minute period

Level

1-2

What to Look For

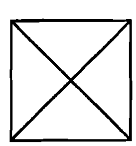
Students should be able to create a wide variety of patterns using the eight prisms. They should be able to give a definition of "pattern" and see that a pattern does not change when it is rotated. Observe the students' approach to the problem and later interview them to determine how they were able to determine variations. Do they self-correct? Do they draw on previous experience?

Anecdotal Notes

Age: 7 years, 4 months

Grade: 2

The student had had experience working with manipulatives in First and Second grade. The teacher first made sure that the student understood the task. This student had some difficulty because she interpreted each face of the block to be a "prism." Since each block has five faces, she saw that it was impossible to choose only eight. The teacher encouraged her to read the directions again more carefully. Once the student understood the directions, the first thing she did was to copy the example. On her first attempt she used only four blocks, making a design that looked like this:



While working with the prisms she frequently talked to herself about what she was doing and tried to get the teacher to more clearly define the task.

Teacher: (Referring to the four-prism pattern the student has just made.) Look at that carefully. Is that the same as the one in the sample?

Student: Yeah...Oh, except it doesn't have the little box around it.

T: Can you figure out a way to make it like the sample?

After several minutes of exploration the student figured out that she could add a prism to each side of the square to make a larger square.

S: (Re-reading the directions.) How many ways can you vary the square... hmmm...like this?

T: It is up to you.

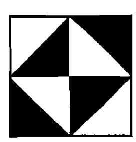
S: Do you have to use all the blocks to make a square?

T: I don't know. What do you think?

S: (Spends several minutes trying to make a square shape from various numbers of prisms.) Yup, you need eight.

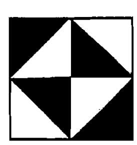
T: Hmmm.

Student spends several minutes manipulating the blocks into this pattern.



Teacher rotates the square one quarter turn.

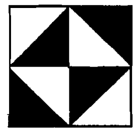
T: Is this the same pattern now?



S: Yes.

Teacher rotates the square another quarter.

T: How about now. Is this the same pattern now?



S: No. (Moves around the table to the other side.) Only if you look at it from this other side.

T: Oh, I'm confused. You said that this first one (the quarter turn) was the same pattern.

S: Only if you look at it from over here. (Cocks her head to look at it from an angle.) The bottom has to always be here. (Looks confused.)

T: I was just asking you what you thought. I don't think there is a correct answer to that.

S: Should I stick with all the blocks or can I use less?

T: Can you make a square with less blocks?

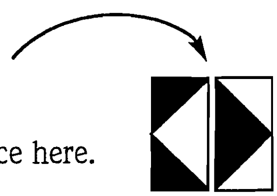
S: Hmmm.

T: Can you make a square with five blocks?

S: (Works on it briefly.) No...I can make a square with six blocks.

T: Let's see it.

S: There. But it has a little space here.



T: When you have that little space there, do you consider it a real square or not?

S: No. If you are supposed to cover all the space, it doesn't work. No! It doesn't work.

T: What about seven blocks. Do you think seven will work for a square?

S: No.

T: You didn't try it and you said no. Why don't you think it will work with seven?

S: I'm pretty sure. But we can do it with eight. You can do it with...hmmm...two!

T: How can we know how many prisms are needed to make a square?

S: It could go like this (pointing to squares): two plus two plus two plus two equals eight.

T: What if we were to go on with this problem...we know we can make a square with two. We know we can make it with eight. What do you think would be the next number we could make a square with?

S: (Thinks briefly.) 16!

T: You don't think it would be 10?

S: Nope. I have to use some more blocks to show you.

T: That's okay. We are getting away from the pattern problem but this is a very interesting problem also.

Student spends some time building another square.

T: So this is the next size that makes a square, huh?

S: Oh...I bet there are others but I just didn't feel like making them.

T: How many prisms is this?

S: (Counts.) Hmmm...18. Hmmm.

T: Is there any other number of prisms between eight and 18 that you can make a square out of?

(Student works on it for a while.)

S: Hmmm... No!

T: That's interesting!

S: We could pile them on top. (Builds a second eight-prism square on top of the first one. Looks at it. Measures it with her finger.) That's not right. It is longer that way. It is too tall.

T: What if it wasn't taller. What if it was only this tall (points). What would we call that shape?

S: Ah…I know it. I forget. Don't tell me. Mmmm…A cube!

T: If you wanted to know for sure what the difference was, what could you do? You are doing it with your eye here but you could do it another way…

S: Hmmm…Measure it. But I don't want to now.

T: Right. That's fine. You know what…we still didn't get back to our first problem. That was how many color designs or patterns can you make. We went off into this other problem because it was so interesting.

S: Should we go back to the other one.

T: Sure.

S: We have to go back to eight prisms. Do you know the answer to this?

T: No, I don't really know the answer.

S: It's big. I don't think I can really find the answer.

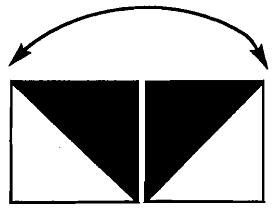
T: I just want to see how many different patterns you can do.

Student continues to make designs. She creates them by first making a diamond shape and then adding four more prisms to square it off. At this point she has created eight different designs. For record-keeping purposes the teacher has copied them onto graph paper. The student has begun to use this as a reference.

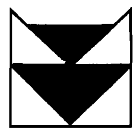
S: There aren't any more ways.

T: You don't think so?

Student continues to explore with the prisms. She creates two two-prism cubes and puts them together:



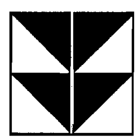
Then adds three more prisms.



S: This one almost works.

T: Think about that one a little more.

She creates another square pattern.



S: I like that one.

T: Mmmm...There is something interesting about this one that you made compared to the other ones you made.

Looks at it for a minute.

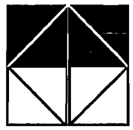
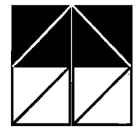
S: I see it too. All the other ones had this in the middle (outlines the diamond). But this one doesn't.

T: That's what I was noticing too. But is it still a square pattern?

S: Yes!!

She creates another–the inverse of the last one she made. Then she creates three more patterns very quickly. At this stage she is still using trial and error to configure shapes that do not square up. She does, however, self-correct immediately.

Student creates the same color pattern that she had done previously but with different prism orientations.

T: Would you say that this  is the same pattern as this one? 

(Teacher shows student sketch of previous pattern.)

S: Different…Although they are just the same color.

T: If this pattern was made out of cloth instead—say you were making a quilt and you were making lots of these—would you put this kind (points to first pattern) and this kind (points to second) in the quilt together?

S: I would do them all like this with the diamond in the middle.

T: I was just looking for your opinion on that.

Student makes more patterns. Now she begins to work very quickly, just changing the color or orientation of one or two prisms at a time. She does not take the whole pattern apart each time.

T: Aren't you clever! How many possible ways do you think there might be of changing this pattern?

S: Lots!!

T: Have you had enough?

S: Okay.

T: I noticed at the end that you got very fast.

S: Look. (She changes just one piece.)

T: Yup. You just made little changes, huh?

Some Final Thoughts

The lines between a guided lesson, an observation, and an interactive interview are gray. Here, while taking notes and taping the discussion, the teacher interacted with the student in a limited way. She encouraged the student to continue by asking probing questions. This helped the student make a number of discoveries for herself.

During this session the student discovered the difference between a prism and the face of a prism and discovered that a square shape can only be made with specific numbers of prisms. She discovered the concept of orientation of pattern and investigated this. She was able to project a mathematical formula and used estimating skills. This child demonstrated an understanding of the properties of a cube, reviewed an application of measurement, and investigated both color and shape symmetry.

ANGLES AND POLYGONS

Make a shape with all the angles fitting together.
Try to make a closed shape. See if you can build an octagon, a hexagon, or a pentagon.

Context and Connections

In order to address this task, students will need a basic understanding of the vocabulary of polygons. They will also need to know the definition of an angle. For younger students, these ideas can be explained simply and in context as the activity is introduced. Older students can use this activity to practice and review the concepts.

What This Task Accomplishes

This exploration can be approached from various levels of understanding. Younger students can use it to explore the notion of polygons with various numbers of sides. Older or more sophisticated students will arrive at the understanding that there is a relationship between the size of the angles and the length of the sides in a polygon.

Tips

Allow students to interpret this task as they choose. The way they interpret the task will say something about their level of understanding. Encourage them to make their own decisions and, ideally, to verbalize their reasons for making these decisions.

Materials

Pyramid set of LMW blocks
Drawing paper and pencils
Cardboard, scissors, protractor (optional)

Time Required for Task

One 45-minute period

Level

K-2

What to Look For

Each student, according to his/her developmental stage, will either decide to incorporate or ignore the size of the angle in addressing this task. If the student incorporates the size of the angle, this a highly challenging task. The student who chooses this route should be encouraged to create a new piece or set of pieces that would allow more solutions. Encourage this child to articulate why it is so difficult to make polygons that "work."

Anecdotal Notes

Age: 5 years, 10 months
Grade: K

In the following observation, the task was adapted to the developmental stage. Initially, the teacher invited the student to explore the materials. He examined the blocks, piling and balancing them into a structure that was partly symmetrical, partly asymmetrical, and very sculptural. Announcing that he has an idea, the student begins to fit the angles on the ends of the blocks together. Although he could not explain his idea, he fit the angles on the ends of the blocks together as he sees their connection to each other. He creates a structure similar to his first one, but with the angles taken into account. This child seems to be clearly ready for this next task. The teacher asks the student to create another structure with "the angles together."

Student: You gave me an idea. (He reorients a series of parallel blocks in a V formation, laying the blocks flat on the table until they fit together on both their sides and edges. The two smallest blocks don't fit at first, so he moves them around until they fit in a V shape.)

S: This is like a hockey playing thing.

Teacher: A goal?

S: Yes!

The teacher asks the student to create a closed shape.

T: Do you know what a closed shape is?

He nods, pauses, taps the table.

T: Thinking about it?

He nods his head again. Next, the student creates a fence and tries to fill it in completely with the rest of the blocks in the set.

S: There aren't enough.

The teacher explains that it's not necessary to fill in the shape, so the student begins to elaborate on his existing closed shape, adding a rectangular entrance way onto his multi-sided polygon in addition to a gate of sorts.

S: Something goes in, something goes out—a cow, bull, or oxen. I want an oxen to go in there. (He laughs at this idea.) This is decoration. A waterfall goes in there down at the end.

T: I want you to build a shape with eight sides.

S: Like how...this is a side? (He counts the sides, but they are just standing randomly, so the teacher points this out to him.) Oh, does it have to be closed?

Then the student builds a fence with four sides and fills it in with four more, then four more again. After some discussion, he realizes what's wrong, and he revises his structure so that it has eight sides, although a couple of the sides are actually one line segment fashioned from two connected blocks. He does not concern himself with precisely lining up the angles in his response.

T: Was that challenging or easy?

S: Easy. (He arranges the pieces back in puzzle form.) There!

Some Final Thoughts

By the end of this session the student understands the concepts of symmetry, structure, and shape. The student is now ready for subsequent lessons on closed shapes and their features. It is interesting to note that the student relates each of his solutions to objects in the real world. His real-world knowledge seems to inform his spatial understanding.

RECTANGULAR SOLID

Put the notched pieces together to form a rectangular solid by standing all of the notched pieces on edge.

Context and Connections

Students have been studying solid geometric shapes. They have just been introduced to the term and concept of a rectangular solid. They have had experiences working with a variety of puzzles and manipulatives and have already had time to use the Stackbuilder in other ways.

What This Task Accomplishes

Students use their problem-solving and reasoning skills to discover combinations of ways to create a rectangular solid. Most students will discover that the Stackbuilder pieces need to be paired and that each pair equals a common length. Some students will make the connection with math fact families, measurement units, or patterns. Most students will see the equality relationship.

Tips

Students who have difficulty creating a rectangular solid might be given the outline of the rectangle on a sheet of paper and asked to build the piece within the outline. If some students continue to have difficulty they should be encouraged to use the box as a frame for their rectangular solid. In order to solve this task, students must discover the fact that common lengths are needed. This common length, or equality, could be used to teach math fact families. This task could also be used in a lesson about patterns or area. Students might be encouraged to lay the Stackbuilder components flat and solve the problem in this manner.

Materials

LMW Stackbuilder block set

Colored pencils, crayons, or markers and graph paper for illustration and documentation

Time Required for Task

One 30-minute period

Level

K-2

What to Look For

Because the Stackbuilder's edges are colored, once the student discovers the concept of common pairs, s/he should not have difficulty creating a rectangular solid and then moving the components around to create other variations of the rectangular solid. S/he may, however, have some difficulty finding all the possible solutions. Look for systematic thinking in discovering all of the solutions.

Anecdotal Notes

Age: 7 years, 3 months

Grade: 2

This student has auditory processing difficulties. Unlike most children, while working with the materials he did not talk to himself or do any vocalizing except when prompted by the teacher. He has had experience working with manipulatives and was given some free time to explore before the task of creating a rectangular solid was explained to him. The task needed to be explained several times. This student needed help with the terms rectangular *and* solid. *Pre-teaching of these terms was done.*

The student worked silently for about five minutes. He placed the pieces on their sides and moved them around, creating several closed shapes. Eventually he created a rectangle but did not use all of the pieces or place them on edge.

Teacher: Are these pieces resting on their edges?

Student: Oh...

T: Do you think there is a way to do it?

S: Yup. I'll make it.

Student works for five or six minutes trying to solve the task by creating a rectangular shape with four of the pieces and then trying to fill it in with the other pieces. He is unsuccessful.

T: Mmmm...Do you think you need these other pieces?

S: Oh...I might know how to do it.

Works for several more minutes. Then becomes discouraged.

S: I don't know how to do it.

T: Keep trying.

Begins to use the pieces on edge and discovers that he can make "trains" of equal lengths by pairing them. He begins to put the pairs next to each other to make a rectangle.

T: Is that it?

S: Yup. That's it. (Has a satisfied look on his face.)

T: Ahhh! That took lots of thinking, huh?

Some Final Thoughts

This child's auditory processing problem is clearly impacting his ability to communicate. It may also have been the reason he didn't know the meaning of the words *rectangular* and *solid.* Once the teacher realized that he didn't know all the words, she retaught them. Given a task such as this, most children will talk to themselves or to the teacher.

When the teacher saw the student was becoming discouraged with the task, she gently cued him by asking if he might need the other pieces and encouraged him to keep trying. In this case, this was all that was needed in order for the child to stick with it and finally feel that he had succeeded. The teacher, of course, will note the difficulties the child had with language and that he needed encouragement to continue.

EXAMPLES OF STUDENT ASSESSMENT: Drawing and Photographs; Record-keeping

On the following pages you will find samples of how one Kindergarten teacher was able to keep a record of her students as they either played freely or tried to do a specific task with the blocks. Students were asked to document their task by making an illustration. Several of these illustrations are included on the following pages.

By using a group observation sheet, the teacher is free to walk around the room and make multiple observations while the students work independently with the blocks. She can document her observations, see what the student knows, and plan for future needs or next steps.

Sam made both illustrations #1 and #2. In the first illustration, the teacher sees that the child is able to create an "AB" pattern and notes that she would like to try a more advanced pattern with him in the future. In illustration #2, she has asked him to do something more difficult, to make a square-shaped pattern. She finds that he has difficulty with this. He was able to make an L shape. His illustration reflects the difficulty he was having. Her next steps are to see if he can make an "ABC" pattern and to have him try the square-shaped pattern again at a later date.

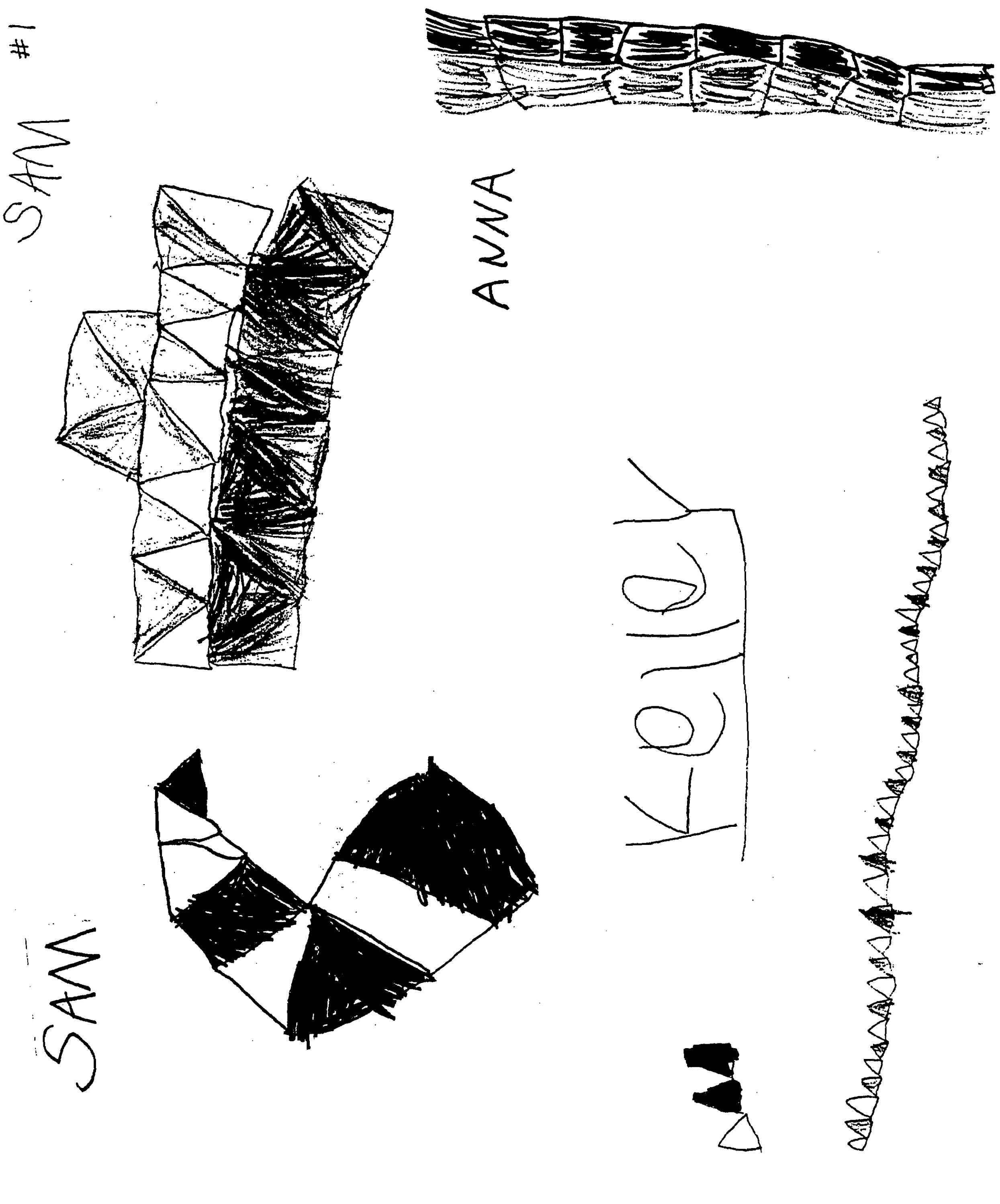
While working with Prismatic, Anna created a paired "AB" pattern and was able to discuss it with the teacher. The teacher notes this and plans to assess other patterns in the future. Kelley used the Prismatic to make an "AB" pattern also. She demonstrated her understanding in a way different from Anna's. She lined the prisms up in a patterned row and then illustrated it. She, also, was able to clearly communicate her thinking to the teacher. The teacher's next step with Anna will be to assess more complicated patterns.

Teacher Observation Sheet

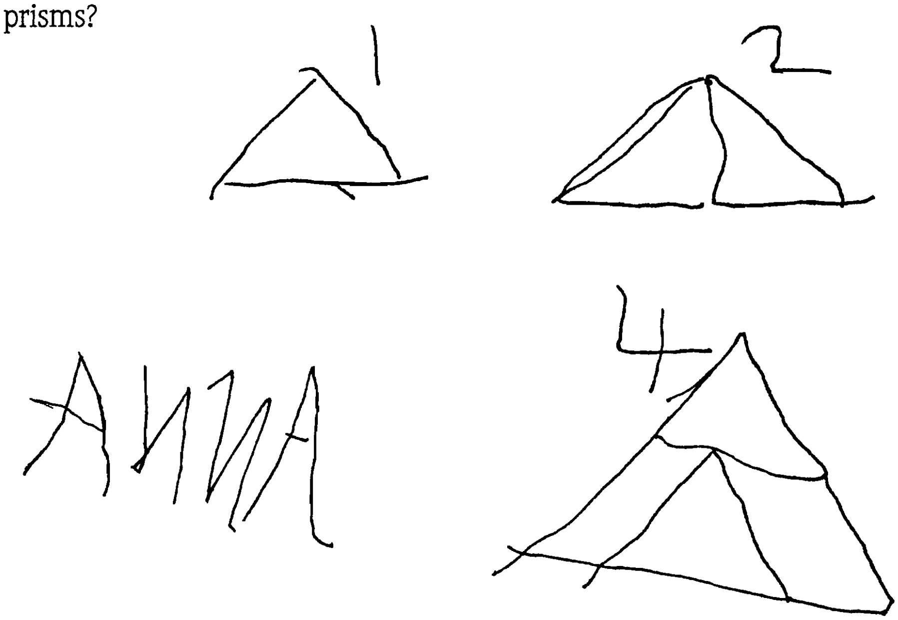
	Name/Date	Observation	What Student Knows	Next Steps
	Hannah 1/6	Using pyramid, made a flat spiral shape	spatial sense	- Use other materials to make spirals - Use the term spiral
Illustration #1	Sam 1/6	I asked Sam if we would make another pattern on top of the one he had started. He made another AB pattern. I asked him to draw a picture of it.	- AB patterns - communication - able to draw an accurate representation of his work - problem solving	Ask Sam if he can make a different kind of pattern
	Kristi, Kelley, Hannah, Kayla & Jessica 1/5	Used pyramid and arcobaleno to make a 2-dimensional "house with rooms" for the teddy bear counters - dramatic play with counters	- how to play cooperatively in sustained dramatic play - spatial sense - communication	see if this is typically how they use these materials
Illustration #3	Anna 1/6	With prismatic, created a vertical AB pattern - sketched it	- knows AB pattern - communication - accurate drawing	Assess other patterns

Teacher Observation Sheet

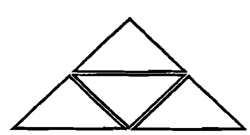
	Name/Date	Observation	What Student Knows	Next Steps
Illustration #2	Sam 1/8	I asked Sam if he could make a square-shaped pattern and draw a picture of it. He made an L-shaped AABB pattern with prismatic	- Knows AABB pattern - communication - problem solving	- Check to see if able to do ABC pattern - Try square-shaped pattern again
	Jessica and Angela 1/8	Lined up prismatic across table - said "Let's count." Counted into 20's - skipped around in 20's	- Both counted accurately to 20.	opportunities to count objects past 20
Illustration #4	Kelley 1/8	Using prismatic made horizontal AB patterns Recorded pattern	- Knows AB pattern - communication - accurate illustration	Assess other patterns
	Hannah 1/8	With arcobaleno made a 3-dimensional symmetrical design	geometry and spatial designs	- Provide more opportunities to make symmetrical designs - Use the term symmetrical


SAM
SAM #1
ANNA

Much can be learned from a student's visual interpretation of a block structure. Anna is a 5-year-old Kindergartner. Consider Anna's drawing of her solution to the Prismatic task: Can you make bigger triangles from the smaller triangular prisms?

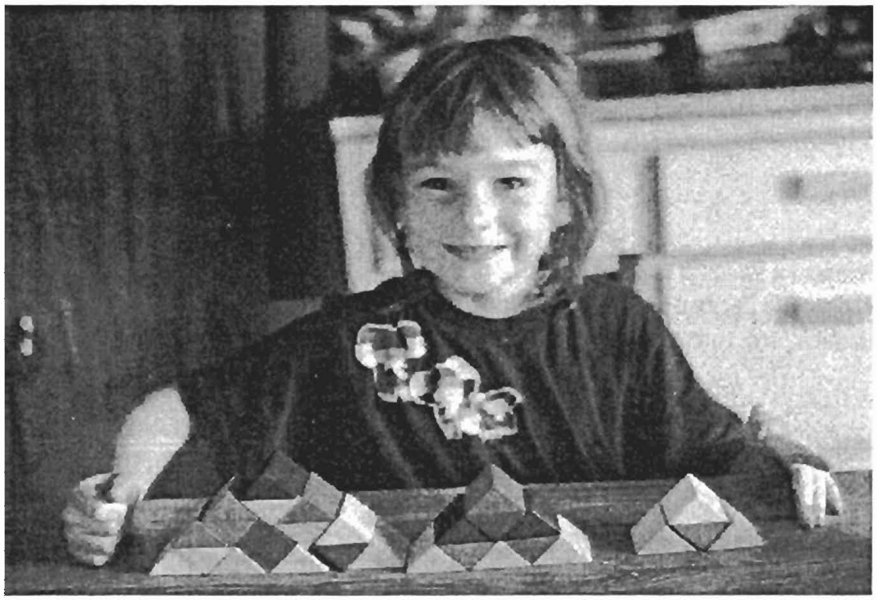


Anna's representation shows that she is able to both solve the problem and represent her solution in the form of a drawing. For a student of her age, this is a sophisticated response. You will note that one of the triangles has been turned around so that it is upright. It should look like this.



At the developmental stage of a kindergartner, a triangle is often only a triangle if the point faces up, so this reorientation is a natural misinterpretation. Anna is also able to label her structure with the correct number of prisms that were used to build it.

A photograph can also be used to capture a significant solution to a task. This 5-year-old Kindergarten student has just completed the same task that was presented to Anna: Can you make bigger triangles from the smaller triangular prisms?

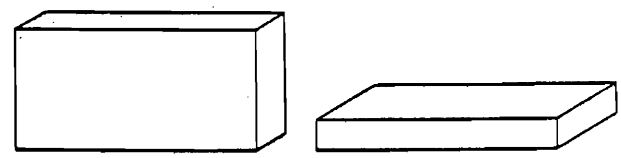


In this photograph, we see that the student is able to reproduce her structure—not only its corresponding shapes, but also the corresponding color patterns. This ability to reproduce a solution is very sophisticated. Often a child will come upon a geometric solution through trial and error; it is quite another thing to be able to reproduce a specific solution successfully a second and a third time.

Another interesting point is that the student has built her triangular structures standing upright. This is a more complex solution than laying the blocks on their triangular faces because it involves building the structure in a specific sequence that requires the center pieces to be buttressed by those on either side.

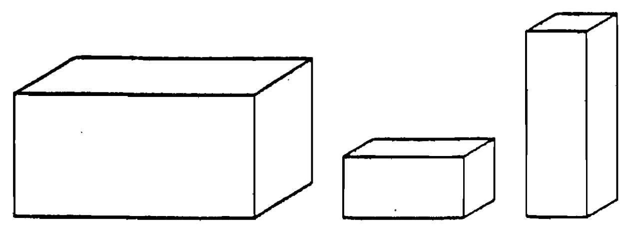
All shapes are **Polyhedra,** three-dimensional solid shapes with flat faces. There are five general categories:

Rectangular Prisms: Prisms have two opposite faces that are the same size and shape (congruent). All other faces, connecting these two opposite faces, are rectangles. In **rectangular prisms**, the two opposite faces are rectangles, so all six faces are rectangles. You can also call these shapes rectangular solids.



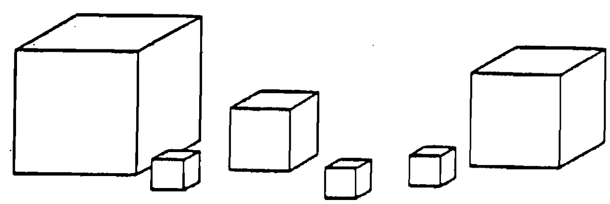
Rectangular prisms

Square Prisms: These are a special kind of rectangular prism. They have two opposite faces that are congruent squares. The other four faces are rectangles.



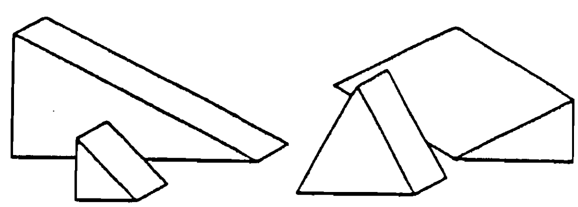
Square prisms

Cubes: Just as the square is a special kind of rectangle, the cube is a special kind of rectangular prism in which all the faces are squares.



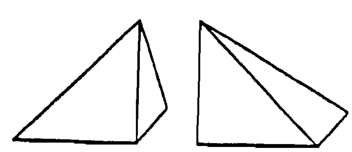
Cubes

Triangular Prisms: These prisms have two opposite faces that are congruent triangles. As in any prism, the faces that connect this pair are all rectangles.



Triangular prisms

Pyramid: Pyramids have one base, which can be any kind of polygon. The rest of the faces are triangles that meet in a single point (vertex.) The pyramid below is a square pyramid. It has a square base and four triangular faces.



Pyramids